A HYBRID MIMO PHASED-ARRAY CONCEPT FOR ARBITRARY SPATIAL BEAMPATTERN SYNTHESIS

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ABSTRACT

Multiple-input multiple-output (MIMO) radar is a multiple aperture technology characterized by the ability to transmit diverse signals at each aperture. This is in contrast to traditional phased-array radar whereby a single signal is transmitted with a phase shift applied at each element to enable steering of the transmit beam. The hybrid MIMO phased-array radar (HMPAR) concept is an outgrowth of the monostatic MIMO construct, in which all sensors have the same view of the far-field target. In the HMPAR, the full transmit array is partitioned into sub-arrays which can be electronically steered in different directions and driven by separate transmit waveforms; furthermore the configuration of the array into sub-arrays can be changed. Here we explore the variety of transmit beampatterns that could be achieved using such a system.

Index Terms—multiple-input multiple-output, beamforming, radar

1. INTRODUCTION

Within the emerging multiple-input multiple-output (MIMO) radar community, there are two fundamental research foci depending on whether the MIMO problem is framed as one involving spatially distributed sensor assets, or collocated sensor assets. MIMO radar has the flexibility of individual signal selection at each aperture, an advantage over traditional phased-array radar systems. This extra degree of freedom poses an enormous challenge in terms of optimizing the expanded trade-space for performance improvements (see [1] and the extensive list of references contained therein).

The focus of this paper is on a notional concept to add another degree of freedom to the MIMO radar trade-space for the collocated asset problem formulation. This concept, which we term the hybrid MIMO phased-array radar (HMPAR), allows for the combining of elements of traditional phased-array radar operation with that of newer MIMO signaling strategies. The HMPAR consists of MP transmit and receive (T/R) elements, organized into M sub-arrays of P elements each. Figure 1 illustrates this principal for the P elements arranged into M rectangular sub-arrays.

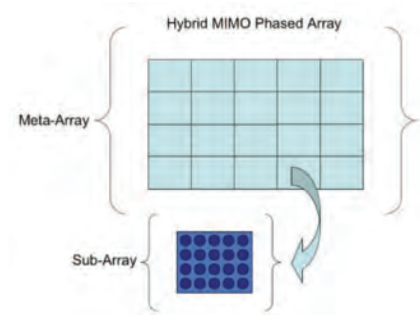

Figure 1: HMPAR Notional Concept

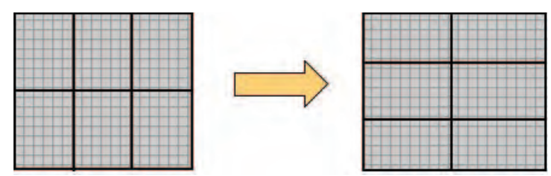

Figure 2: Reassignment of Subarrays

Each sub-aperture of the array has the ability to independently choose and, through the use of passive phase-shifting, arbitrarily steer the transmit beam towards a region of interest. Such a system could be located on an airborne or ground-based platform and could lead to performance improvements by offering the end-user the ability to conduct multiple simultaneous radar modes (*i.e.*, search and track, etc.). The objective of current and proposed research is to identify transmit signaling strategies and adaptive receive signal processing methodologies consistent with the requirements of the HMPAR.

2. TRANSMIT BEAMPATTERN SYNTHESIS PROBLEM

In [2], a description of the beampattern synthesis problem from the viewpoint of a uniform linear array (ULA) is given. For the narrowband case (assume the chip length is greater than the transmit time across the array face), half-wavelength element spacing (to preserve the 1-to-1 relationship of the electrical and physical angles), individual element impedance of 1Ω, and mutual impedance of 0Ω

978-1-4244-3677-4/09/$25.00 ©2009 IEEE

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **JAN 2009**	2. REPORT TYPE	3. DATES COVERED **00-00-2009 to 00-00-2009**

4. TITLE AND SUBTITLE **A Hybrid MIMO Phased-Array Concept for Arbitrary Spatial Beampattern Synthesis**	5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Air Force Research Laboratory,Wright-Patterson AFB,OH,45433-7333**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S) 11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

See also ADM002264. Presented at the IEEE Digital Signal Processing Workshop (13th) and Signal Processing Education Workshop (5th) Held in Marco Island, Florida on 4-7 January 2009. Sponsored by ONR.

14. ABSTRACT

Multiple-input multiple-output (MIMO) radar is a multiple aperture technology characterized by the ability to transmit diverse signals at each aperture. This is in contrast to traditional phased-array radar whereby a single signal is transmitted with a phase shift applied at each element to enable steering of the transmit beam. The hybrid MIMO phased-array radar (HMPAR) concept is an outgrowth of the monostatic MIMO construct, in which all sensors have the same view of the far-field target. In the HMPAR, the full transmit array is partitioned into sub-arrays which can be electronically steered in different directions and driven by separate transmit waveforms; furthermore the configuration of the array into sub-arrays can be changed. Here we explore the variety of transmit beampatterns that could be achieved using such a system.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**5**	

(each transmitter will transmit at full power) the normalized power density, $P(\theta,\phi)$ (watts/steradian), is the synthesized beampattern of interest and is shown to be [2]

$$P(\theta,\phi) = \frac{1}{4\pi}\mathbf{a}^T(\theta)R\mathbf{a}^*(\theta) \tag{2.1}$$

where the factor $\frac{1}{4\pi}$ assumes isotropic radiation sources. The direction vector

$$\mathbf{a}(\theta) = \left[e^{j\psi_1(\theta)}, e^{j\psi_2(\theta)}, \ldots, e^{j\psi_M(\theta)}\right]^T \tag{2.2}$$

is controlled by the electrical angle ψ given by

$$\psi_i(\theta) = \frac{2\pi d_i}{\lambda}\sin\theta \tag{2.3}$$

with the subscript, i, denoting all elements from 1 to M. Additionally, the traditional phased-array beampattern is found by substituting the cross-correlation matrix, R, with a rank 1 matrix, thus

$$P(\theta,\phi) = \mathbf{a}^T(\theta)\mathbf{a}^*(\theta) \tag{2.4}$$

where the superscripts T and $*$ denote transpose and complex conjugation, respectively.

For the general MIMO ULA case, all transmit signals are assumed orthogonal, resulting in a cross-correlation matrix that is equal to the identity matrix

$$R = I\,. \tag{2.5}$$

However, [3] details the difficulty in achieving the orthogonality condition for all nonzero-delays. Thus, all transmitted signals for the general MIMO ULA case would exhibit some degree of cross-correlation. This is not necessarily detrimental, depending on the formulation of the beampattern synthesis problem. For the HMPAR concept, quasi-orthogonal signal sets are combined with, independent sub-array electronic steering, to yield arbitrary beampatterns with more flexibility than in [2,4]. Thus, the problem formulation centers around the correct choice of the cross-correlation matrix, for the case of a traditional phased-array radar, with the composite array manifold given by $\mathbf{a}(\theta)$, also being user-defined, for the case of the HMPAR concept.

3. TRANSMIT BEAMFORMING FOR A TWO-DIMENSIONAL HMPAR

In the previous section, the transmitted beampattern was determined by both the array response at angle θ, $\mathbf{a}(\theta)$, and the cross-correlation matrix R. For M signals $s_1(t)\ldots s_M(t)$ with zero lag correlation

$$R(0) = \int_{-\infty}^{\infty}\mathbf{s}(t)\mathbf{s}^H(t)dt \tag{3.1}$$

the energy spectral density (*i.e.*, the distribution of transmitted signal energy in space) is given by

$$S(\theta) = \mathbf{a}^T(\theta)R(0)\mathbf{a}^*(\theta)\,. \tag{3.2}$$

This approach to beampattern synthesis is capable of both omnidirectional and spotlight phased-array performance, based upon the cross-correlation matrix, R. If, $R(0)$ is a rank-one matrix of the form

$$R(0) = c\mathbf{a}^*(\theta_0)\mathbf{a}^H(\theta_0) \tag{3.3}$$

then all the signals are perfectly correlated and the transmit beam is formed pointing in the direction of θ_0. This would correspond to the spotlight phased-array performance, useful for a MIMO radar operating in tracking mode.

If, the cross-correlation matrix does not conform to either of these two extremes, then we are left with partially correlated waveforms; useful for arbitrary beampattern synthesis where the objective is to distribute transmit energy over some defined range swath of interest between the two previously described extreme regimes. The remainder of this section details the development of a closed-form method for selecting signals that result in approximately rectangular transmit beampatterns for MIMO radars with uniform linear arrays (ULA), and later, rectangular transmit arrays.

For the case of a ULA with a single angular parameter (the electrical angle ϕ), we define a set of signals in which the beamwidth is controlled by a single scalar parameter α. Let $s_i(n), i = 1\ldots M,\ n = 0, N-1$ be given by the expression

$$s_i(n) = e^{\frac{j2\pi\alpha(i-1)n}{N}}e^{\psi(i-1)} \tag{3.4}$$

with the expression for ψ being

$$\psi = \pi\alpha\left(\frac{N-1}{N}\right). \tag{3.5}$$

(3.5) denotes a value of ψ that will steer the center beam toward array broadside.

The cross-correlation between signal i and signal k is

$$r_{ik} = \sum_{n=0}^{N-1}s_i(n)s_k^*(n) \tag{3.6a}$$

$$= \sum_{n=0}^{N-1}e^{\frac{j2\pi\alpha(i-1)n}{N}}e^{\frac{-j2\pi\alpha(k-1)n}{N}}e^{\psi(i-1)}e^{-\psi(k-1)} \tag{3.6b}$$

$$= \sum_{n=0}^{N-1}e^{\frac{j2\pi(i-k)\alpha n}{N}}e^{\psi(i-k)} \tag{3.6c}$$

$$= e^{\psi(i-k)}\left(\frac{1-e^{j2\pi(i-k)\alpha}}{1-e^{j2\pi(i-k)\alpha/N}}\right) \tag{3.6d}$$

$$= \frac{\sin\pi\alpha(i-k)}{\sin\pi\alpha(i-k)/N}\,. \tag{3.6e}$$

The full cross-correlation matrix $R(0)$, with ik element r_{ik}, as given above, is a Toeplitz matrix with the *lth* diagonal given by the Dirichlet function in (3.6e). For α =1, $R(0)$ is N times the identity matrix; and when α =0, $R(0)$ is N-times the rank-one all-ones matrix. These correspond to the two

extremes of quasi-orthogonal and spotlight phased-array signaling. For values of $0<\alpha<1$, the resultant beampattern is approximately rectangular, where the beamwidth in electrical angle space is proportional to α.

For a ULA, with the same given assumptions as used for the derivation of (2.1), the array response vector is

$$\mathbf{a}(\theta)=\left[1, e^{j\phi}, \ldots, e^{j(M-1)\phi}\right]^T \tag{3.7}$$

where ϕ is the electrical angle that depends on the center frequency and array spacing. Assuming ϕ covers the range of $[-\pi,\pi]$, then the quadratic form (3.2) becomes

$$S(\phi)=\sum_{l=-M}^{M}(M-|l|)\left(\frac{\sin\pi\alpha l}{\sin\alpha\pi l/N}\right)e^{-j\phi l} \tag{3.8}$$

where the double sum has been converted to a single sum over the diagonals of $R(0)$. (3.8) has the form of the Fourier transform of a triangular windowed Dirichlet function. Since the Fourier transform of the unwindowed Dirichlet function is a rectangle in ϕ-space, the transmit beampattern $S(\phi)$ is the convolution of a rectangle with a sinc-squared function, the latter being the Fourier transform of the triangle function described by $M-|l|$. The nominal one-sided beamwidth in ϕ-space is $\pi\alpha$.

Extending this to the case of a rectangular array, as in the case of the HMPAR, is straightforward. Suppose that we arrange the M transmitters in an $M_1\times M_2$ uniformly spaced grid, with $M_1M_2=M$. We denote the signal row and column indices for the corresponding transmitter with i_1 and i_2, respectively. For convenience, each signal is written in terms of a row vector $\boldsymbol{s}$, with the n notation suppressed. The signal $\mathbf{s}_{i_1,i_2}$ is the Kronecker product of two complex exponentials, as given in (3.5)

$$\mathbf{s}_{i_1,i_2}=\mathbf{s}_{i_1}\otimes\mathbf{s}_{i_2}, \tag{3.9}$$

with $\otimes$ representing the Kronecker product. The lengths of $\mathbf{s}_{i_1}$ and $\mathbf{s}_{i_2}$ are each $\sqrt{N}$, instead of N. However, note that the length of each signal is N samples, as before.
The correlation of $\mathbf{s}_{i_1}$ and $\mathbf{s}_{i_2}$ is thus

$$r_{i_1i_2k_1k_2}=\mathbf{s}_{i_1}\mathbf{s}_{k_1}^H\mathbf{s}_{i_2}\mathbf{s}_{k_2}^H. \tag{3.10}$$

The full correlation with entries, as in (3.10), is the Kronecker product of two Toeplitz correlation matrices for the ULA case

$$\mathbf{R}=\mathbf{R}_h\otimes\mathbf{R}_v \tag{3.11}$$

where $\mathbf{R}_h$ and $\mathbf{R}_v$ are the signal correlation matrices for $\mathbf{s}_{i_1}$ and $\mathbf{s}_{i_2}$, respectively, and the subscripts h and v signify *horizontal* and *vertical*, respectively.

In the case of the ULA, $\mathbf{a}(\theta)$, now a function of two electrical angles u and v, separates as the product of one-dimensional response vectors

$$\mathbf{a}(u,v)=\mathbf{a}_h(u)\otimes\mathbf{a}_v(v). \tag{3.12}$$

The quadratic form describing the transmit beampattern is thus

$$S(u,v)=(\mathbf{a}_h(u)\otimes\mathbf{a}_v(v))^T(\mathbf{R}_h\otimes\mathbf{R}_v)(\mathbf{a}_h(u)\otimes\mathbf{a}_v(v))^* \tag{3.13}$$

$$=\left(\mathbf{a}_h^T\mathbf{R}_h\mathbf{a}_h^*\right)\left(\mathbf{a}_v^T\mathbf{R}_v\mathbf{a}_v^*\right). \tag{3.14}$$

A pattern in (u,v) space is the separable products of the one-dimensional patterns in u and v. Further discussion on the concept of $(u,v,)$ space and application of (3.14) to synthesizing arbitrary beampatterns are presented in the next section.

4. DEMONSTRATION OF HMPAR TRANSMIT BEAMPATTERN ADAPTIVITY

We will introduce the scenario used to illustrate the HMPAR concepts through simulation. The planar array will consist of 900 elements arranged as a 30x30 grid of transmit elements with half-wavelength spacing. This will then be partitioned into M sub-arrays of P elements each. In simulations where square sub-arrays are desired, there will be M=25 sub-arrays of P=36 elements. In other cases the meta-array will be partitioned into M=30 subarrays of P=30 elements each.

The two controls that dictate the HMPAR transmit beampattern are the signal correlations and the sub-array electronic steering. Initial investigation yielded no particular advantage to having fanned sub-arrays and correlated signals simultaneously. Rather, there appear to be two distinct modes of the HMPAR:

1. Fanned sub-arrays and quasi-orthogonal signals
2. Focused sub-arrays and correlated signals

The term "fanned sub-arrays" is meant to convey that the individual sub-array steering directions are different and are spread through the search volume of the radar array; this is in contrast to the term "focused sub-arrays," where the conveyed meaning is sub-arrays with commonality in steering directions.

Mode 1 is used for the synthesis of broad beampatterns, ranging from fully omnidirectional, to the spatial beampattern of an individual sub-array. Mode 2 is useful for the synthesis of narrow beampatterns, ranging from that of a signal sub-array to that of traditional phased-array radar.

MATLAB was used for creating the visualization of the two-dimensional beampattern of a 30x30 element HMPAR. The planar array, or meta-array, lies in the x-y plan of a three-dimensional coordinate system. The normal to the meta-array plane is in the z-axis direction, with the rows and columns of the array aligned with the x- and y-axes, respectively. The spherical coordinates in this system are (r, θ, φ), where θ represents the elevation angle with

respect to the z-axis and ϕ represents the azimuthal angle of a point projected onto the x-y plane.

Beampatterns could be expressed as a function of azimuth and elevation (θ, ϕ), however, the use of electrical angles u and v are more convenient for display. The electrical angles are defined as

$$u = \sin\theta\cos\phi \quad (4.1a)$$

$$v = \sin\theta\sin\phi\,. \quad (4.1b)$$

(u,v) represents the projections of a unit vector in the (θ, ϕ) direction onto the x-y plane. Further, a set of element weights $\boldsymbol{w}$ arranged as a matrix, to map onto the array, can be decomposed as a separable product of 1-D weight vectors, as show by,

$$\mathbf{w}=\mathbf{w}_h(u)\mathbf{w}_v(v). \quad (4.2)$$

Thus, in (u-v) space, the horizontal and vertical directions are can be controlled separately. The resultant beampattern is formed by a two-dimensional Fourier transform of the weight matrix.

To better understand the viewpoint of the display, imagine the observer is looking forward into the hemisphere centered on the normal of the array plane, and seeing the beampattern on that hemisphere projected onto the plane. For this reason, the only meaningful angles in (u-v) space are those on the interior of the unit circle. This caveat has been taken into consideration and all areas outside of the unit circle are shown as null. The next section details the results of the initial simulations of the MATLAB code, in addition to, providing concise explanations of the meaning of the displays for various signaling strategies. Ever attempt has been made to provide results that demonstrate the fundamental advantages of the HMPAR concept.

4.1. HMPAR Mode 1: Fanned Sub-arrays and Quasi-Orthogonal Signals

The first two scenarios will focus on the fanned sub-arrays with quasi-orthogonal signals, or HMPAR mode 1. Each display figure contains four individual plots. The upper-left display shows the HMPAR array configuration, the upper-right display shows the individual sub-array steering direction in (u,v) space. On the lower-left of the figure is the individual beampattern resulting from a single sub-array, in this case the top far-right sub-array. The entire synthesized beampattern of the HMPAR is shown in the lower-right of the figure. The scale of this beampattern is from zero to maximum, as discussed previously.

4.2.1. HMPAR Transmit Beampattern, Mode 1, Scenario 1
Figure 3 illustrates how the HMPAR could be used to generate a beampattern that illuminates a large square area, thereby, covering a majority of the search volume of interest.

4.2.2. HMPAR Transmit Beampattern, Mode 1, Scenario 2
Figure 4 illustrates how the HMPAR could be used to create a beampattern that illuminates a smaller square area, than in scenario 1. This focuses more of the transmit power into smaller subsection of the search volume, representing the case between omnidirectional and spotlight performance.

4.2. Mode 2: Focused Subarrays and Correlated Signals

These last three scenarios will focus on the focused sub-arrays with correlated transmit signals, or HMPAR mode 2. As before, each display figure contains four individual plots. However, since the sub-arrays are all steered towards a common direction, along boresight, there is no reason to display the upper-right display showing the individual sub-array steering direction in (u,v) space. Instead, the upper-right display shows the sub-array beampattern and the lower-left display shows the beam-pattern for the meta-array, which is the hypothetical array of ominidirectional transmitters at the sub-array phase centers. The upper-left and lower-right displays are the same as in section 4.1.

4.3.1. HMPAR Transmit Beampattern, Mode 2, Scenario 1
Figure 5 shows how the HMPAR can operate as traditional phased-array radar. All signals are correlated, α =1. This figure is the equivalent of exciting and steering 900 elements of a 30x30 element array at half-wavelength spacing.

4.3.2. HMPAR Transmit Beampattern, Mode 2, Scenario 2
Figure 6 demonstrates the utility of the reassignment of the sub-arrays. There are now 30 sub-arrays of 30 elements each, with the meta-array arranged as a 3x10 configuration of sub-array consisting of 3x10 elements each. The value of α is 0.5 and the resulting beampattern shows a narrow vertical beamwidth, while the horizontal beamwidth is much broader. This is consistent, as the beampattern from a ULA would have similar properties, depending in the arrangement of elements.

4.3.3. HMPAR Transmit Beampattern, Mode 2, Scenario 3
Figure 7 again shows another rearrangement of the sub-arrays. This time there are now 30 sub-arrays of 30 elements each, with the planar-array arranged as a 1x30 configuration of sub-array consisting of 30x1 elements each. We would expect the beampattern to have a tight beamwidth in the vertical direction, with a broader horizontal beamwidth (due to the fewer number of horizontal elements contained in each sub-array).

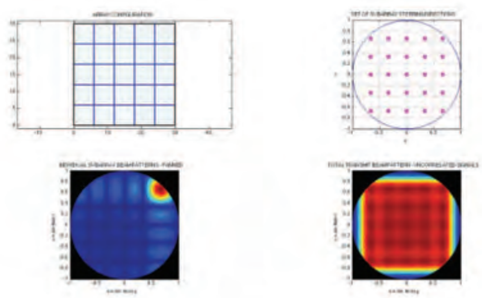

Figure 3: HMPAR, Mode 1, Scenario 1

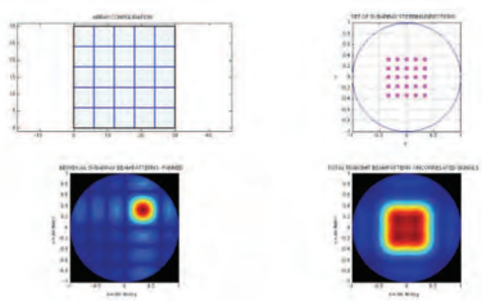

Figure 4: HMPAR, Mode 1, Scenario 2

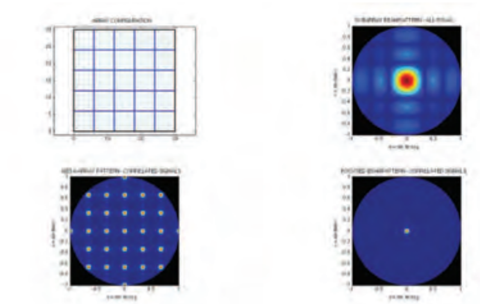

Figure 5: HMPAR, Mode 2, Scenario 1

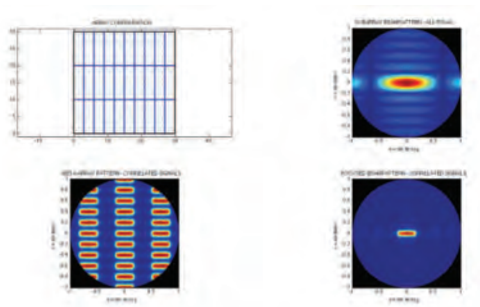

Figure 6: HMPAR, Mode 2, Scenario 2

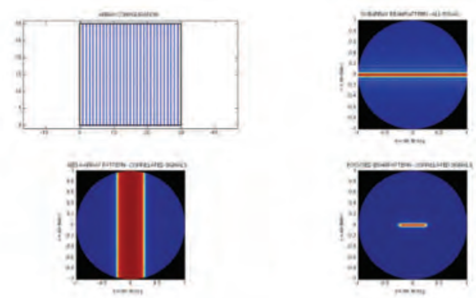

Figure 7: HMPAR, Mode 2, Scenario 3

5. SUMMARY AND CONCLUSIONS

Initial exploration of the HMPAR concept demonstrated a variety of arbitrary transmit beampatterns that could be achieved with such a system. Much important work remains, as the number of open problems identified for full characterization of the HMPAR is quite large. Future publications will focus on the signaling strategies for the HMPAR and the development of the associated MIMO ambiguity function.

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